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Estimation of multi-component stress-strength model based on geometric upper record values

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ABSTRACT
Stress-strength models have special importance in reliability literature and engineering applications. This paper consists of the estimation problem of a stress-strength model with a multi-component system, i.e., a system that can be regarded to be alive if at least s out of k (s $\leq$ k) strength components exceed the stress component. The reliability of such a system has been obtained when both the stress and the strength variables have Geometric distributions. UMVUE of $R_{s,k}$ is obtained based on upper record values. Bayesian estimators under the squared error loss function using the conjugate beta prior distributions have been obtained. A simulation study has been implemented to assess the performance of estimates.

KEYWORDS
Stress-strength, Multi-component, Upper record values, UMVUE, Loss function.

1. Introduction

Record values can be viewed as order statistics, especially closely connected to extreme order statistics, whose size is determined by the values and the order of occurrence of the observations. These record values are more interesting in reliability and survival analysis when the products fail under stress [10]. In this setting, the savings of the cost of the experiment may be considerable if observations are made sequentially and record values are registered. Record values and record statistics are very popular because they arise naturally in many fields of study, like climatology, hydrology, geology, sports, medicine, and so on. The distribution of record values was found in [18] and [21]. It should be noted that the amount of information provided by records is considerable [28]. It has already been proven that upper record values contain more information than the same number of i.i.d. (independently and identically distributed) observations [14]. Another example that Computer Science with a linear search algorithm, where comparisons are made to determine the maximum element in a set.

In statistical inference literature, the renowned stress-strength model, i.e., $R =$

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$P(X \leq Y)$, measures the difference between two populations. This model has various applications in different fields of science, such as reliability, quality control, demand-supply systems, and medicine, etc. In reliability theory, if X represents the applied stress on a component by the operating environment and Y is that component's inherent strength or resistance, then R is the probability that the component performs satisfactorily.

A study on the estimation of R when X and Y are independently distributed geometric random variables was done by [11]. Bayes estimates of the same have been found in [2]. A study on different types of estimation processes of the negative binomial distribution was considered in [8] and [15] in the context of a system reliability. Some studies of estimation of R using Poisson distribution found in [4] and [5]. There is a study of the application of log series distribution in [13]. [6] considered the power-series distributions in this case. E-Bayesian estimation of geometric model using record values was considered in [22].

Some authors have considered continuous distribution in finding the stress-strength reliability R. For example, [16] used the Gompertz distribution. Weibull distribution was used by [14]. Type-II censored data for the Rayleigh distribution was used for finding the Bayes estimate of R, done by [1]. [12] used exponential-Poisson distribution, [9] used generalized exponential distribution, and [7] used inverted gamma distribution to estimate R using the Bayesian method. A few more studies on reliability can be found in [23], [24], [25].

The motivation for considering the geometric distribution is that this type of model can be found, e.g., in hydrology and climatology, while modelling durations of various phenomena, like droughts, cold and warm spells, and others. The number of time intervals during this process remains above or below a level can be modeled using a geometric distribution (see [29] and [30]).

When some prior information regarding the parameter(s) of the distribution is available, one can utilise it in the Bayesian inference study. The Bayes estimate is expected to be more efficient than the classical parametric estimates, as some prior information is incorporated into the former. In this article, we aim to determine the Bayes estimates of the stress-strength reliability (R) under the squared error loss (SEL) function.

Generally, we came across a single stress-strength component system. But in several practical scenarios, we have more than one strength(or stress), where a system has more than one component, each of which has its own strength. Multi-component stress strength models are useful in communication and industrial systems to logistics, and military systems. In this model, there is one stress X and k strength components $Y_1, Y_2, \ldots, Y_k$. A system functions if at least s, $1 \leq$ s $\leq$ k, the strength components are not less than the stress component. In this system, the reliability parameter is $R_{s,k}=$ P{at least s out of $(Y_1, Y_2, \ldots, Y_k)\geq$X}. Some common examples of multi-component systems are "parallel and series circuits". An aircraft generally contains four engines. Aircraft can smoothly run only when at least 2 out of 4 engines work, i.e. 2-out-of-4 component system. An automobile with a V-8 engine that works if at least 4 out of 8 cylinders rung, i.e. "4-out-of-8:G" system. A suspension bridge consists of k number of vertical cable pairs. The bridge would survive if at least s out of k vertical cables through the deck are not damaged.

The application of stress-strength reliability in multi-component systems based on upper record values can be seen in several industrial tests, mainly where most of the systems can't survive when they are under high level of stress. As an example, an electrical power station consists of eight generating units, and a light amount of electricity is generated if at least six generating units are operating. In some experimental tests of energy, these power stations are exposed to very high stress to test their ability to carry out their functions under high level of stress. As a result, most of them are found to collapse immediately, but a few survive for a short period of time, which is then recorded as the first observation of upper record values, if it occurs for a longer period, it will be recorded as the 2nd observation of the sample and so on. In industry and reliability tests, many products may fail to function in an environment of too high temperature. In such experiments to get the precise point, measurements may be made sequentially, and only values larger (or smaller) than all the previous ones are recorded. In those cases, we prefer to use those recorded values for the studies. Estimation of R for geometric distribution using lower record values was studied by [19]. [20] also found the multi-component stress strength reliability model for Weibull distribution.

This article is structured into the following topics. Section 2 details the fundamental model assumptions and the derivation of $R_{s,k}$ and R. Subsequently, Section 3 presents the precise formulas for the Uniformly Minimum Variance Unbiased Estimators (UMVUE) of $R_{s,k}$ and R. Section 4 then focuses on developing the Bayesian estimators for these same quantities. To validate the theoretical results, Section 5 offers findings from a simulation study. Finally, Section 6 provides concluding thoughts on the presented work.

2. Reliability models

Let the stress X and the strength Y_i, $i = 1, 2, ..., k$ be independent random variables such that X$\sim$*Geometric*(θ_1) and Y$\sim$*Geometric*(θ_2). Let $F(x)$ and $G(y)$ be the cumulative distribution function (c.d.f.)s of X and Y, respectively.

$$\begin{aligned} P(X = x) &= \theta_1(1-\theta_1)^{x-1} \qquad x = 1, 2, ... \\ F(x) &= 1-(1-\theta_1)^{x} \qquad x = 1, 2, ... \end{aligned}$$

and

$$\begin{aligned} P(Y = y) &= \theta_2(1-\theta_2)^{y-1} \qquad y = 1, 2, ... \\ G(y) &= 1-(1-\theta_2)^{y} \qquad y = 1, 2, ... \end{aligned}$$

Let $R_{s,k}$ be the reliability of the stress-strength model with one stress X and k strength components $Y_i s'$, $i = 1, 2, \ldots, k$ where the system works if at least k out of s strength components work.

$$\begin{aligned}
R_{s,k} &= P[Y_{s,k} \geq X] \\
&= \sum_{x=1}^{\infty} P(Y_{s,k} \geq x) P(X = x) \\
&= \sum_{x=1}^{\infty} \sum_{w=s}^{k} \binom{k}{w} [1 - G(x-1)]^{w} [G(x-1)]^{k-w} P(X = x) \\
&= \sum_{x=1}^{\infty} \sum_{w=s}^{k} \binom{k}{w} (1-\theta_2)^{xw-w} \sum_{u=0}^{k-w} (-1)^{u} \binom{k-w}{u} (1-\theta_2)^{(x-1)u} P(X = x) \\
&= \sum_{w=s}^{k} \binom{k}{w} \sum_{u=0}^{k-w} (-1)^{u} \binom{k-w}{u} \psi(\theta_1, \theta_2, w, u) \qquad (1)
\end{aligned}$$

where,

$$\begin{aligned}
\psi(\theta_1, \theta_2, w, u) &= E[(1-\theta_2)^{(X-1)(w+u)}] \\
&= \frac{\theta_1}{1 - (1-\theta_1)(1-\theta_2)^{w+u}}.
\end{aligned}$$

For single component stress - strength reliability, $R = \frac{\theta_1}{1-(1-\theta_1)(1-\theta_2)}$.
Let $(R_1, R_2, ..., R_m)$ be first m observed upper record values from *Geometric*(θ) distribution. Then from [26], joint p.m.f. of $(R_1, R_2, ..., R_m)$ becomes

$$\begin{aligned}
P(R_1 = r_1, R_2 = r_2, .., R_m = r_m | \theta) &= P(R_m = r_m) \prod_{i=1}^{m-1} \frac{P(R_i = r_i)}{P(R_i > r_i)} \\
&= \theta^{m} (1-\theta)^{r_m - m}, \qquad 1 \leq r_1 < r_2 < ..... < r_m. \qquad (2)
\end{aligned}$$

By factorization theorem R_m is sufficient statistic for θ and

$$P(R_m = r_m) = \binom{r_m - 1}{m-1} \theta^{m} (1-\theta)^{r_m - m}, \qquad r_m = m, m+1, ... \qquad (3)$$

3. UMVUE of $R_{s,k}$ and R

Within statistical inference, the Uniformly Minimum Variance Unbiased Estimator (UMVUE) stands out as an optimal choice, offering a unique blend of unbiasedness and minimal variance. By balancing accuracy (unbiasedness) and precision (minimum variance), UMVUE emerges as an efficient and highly desirable estimator. So, we obtain the expression of UMVUE of $\phi(\theta)$ as general case in the following result.
Result: For any two integer α and β with $\alpha < m$, UMVUE of $\phi(\theta) = \theta^{\alpha}(1-\theta)^{\beta}$ is

$$\widehat{\phi}(\theta) = \frac{\binom{R_m - \alpha - \beta - 1}{m - \alpha - 1}}{\binom{R_m - 1}{m-1}} \qquad R_m \geq m + \beta. \qquad (4)$$

Proof: Since R_m is complete sufficient statistic for θ, UMVUE of $\phi(\theta)$ can be calculated by solving the following equation [27]

$$\begin{aligned} \sum_{r_m=m}^{\infty} \xi(r_m) P[R_m = r_m|\theta] &= \phi(\theta) \\ or, \sum_{r_m=m}^{\infty} \xi(r_m) \binom{r_m-1}{m-1} \theta^m (1-\theta)^{r_m-m} &= \theta^{\alpha}(1-\theta)^{\beta} \end{aligned}$$

This can be rewritten as

$$\sum_{r_m=m+\beta}^{\infty} [\xi(r_m) \frac{\binom{r_m-1}{m-1}}{\binom{r_m-\alpha-\beta-1}{m-\alpha-1}}] \binom{r_m-\alpha-\beta-1}{m-\alpha-1} \theta^{m-\alpha}(1-\theta)^{r_m-\alpha-\beta-(m-\alpha)} = 1$$

if $\xi(r_m) = 0$ for $r_m = m, ..., m+\beta-1$. By using the completeness property, we get

$$\xi(r_m) = \frac{\binom{r_m-\alpha-\beta-1}{m-\alpha-1}}{\binom{r_m-1}{m-1}}.$$

Finding the UMVUE Let $(R_1, R_2, ..., R_m)$ and $(T_1, T_2, ..., T_n)$ be first observed m and n upper record values from $Geometric(\theta_1)$ and $Geometric(\theta_2)$ respectively. So, T_n and R_m are complete sufficient statistic for θ_1 and θ_2 respectively.
Now,

$$\psi(\theta_1, \theta_2, w, u) = \sum_{x=1}^{\infty} \psi_2(x, \theta_2)\psi_1(x, \theta_1) \tag{5}$$

where, $\psi_2(x, \theta_2) = (1-\theta_2)^{(x-1)(w+u)}$ and $\psi_1(x, \theta_1) = \theta_1(1-\theta_1)^{x-1}$.
By using the expression of UMVUE of $\phi(\theta)$, we obtain the UMVUE of $\psi_2(x, \theta_2)$ for fixed x as

$$\widehat{\psi_2}(x, \theta_2) = \frac{\binom{R_m-(x-1)(w+u)-1}{m-1}}{\binom{R_m-1}{m-1}} \qquad R_m \geq x(w+u)-1$$

and UMVUE of $\psi_1(x, \theta_1)$ for fixed x as

$$\widehat{\psi_1}(x, \theta_1) = \frac{\binom{T_n-x-1}{n-2}}{\binom{T_n-1}{n-1}} \qquad T_n \geq x+n-1.$$

So,

$$\widehat{\psi}(\theta_1, \theta_2, w, u) = \sum_{x=1}^{\min(T_n-n+1, \frac{R_m-m}{w+u}+1)} \frac{\binom{R_m-(x-1)(w+u)-1}{m-1}}{\binom{R_m-1}{m-1}} \frac{\binom{T_n-x-1}{n-2}}{\binom{T_n-1}{n-1}}.$$

Therefore

$$\widehat{R}_{s,k} = \sum_{w=s}^{k}\binom{k}{w}\sum_{u=0}^{k-w}(-1)^u\binom{k-w}{u}\sum_{x=1}^{\min(T_n-n+1,\frac{R_m-m}{w+u}+1)}\frac{\binom{R_m-(x-1)(w+u)-1}{m-1}}{\binom{R_m-1}{m-1}}\frac{\binom{T_n-x-1}{n-2}}{\binom{T_n-1}{n-1}}. \tag{6}$$

And

$$\widehat{R} = \sum_{x=1}^{\min(T_n-n+1,R_m-m+1)}\frac{\binom{R_m-x}{m-1}}{\binom{R_m-1}{m-1}}\frac{\binom{T_n-x-1}{n-2}}{\binom{T_n-1}{n-1}}.$$

4. Bayesian estimation

In this section, the Bayesian estimator of stress-strength reliability P($Y \geq X$) has been found under the squared error loss (SEL) function. Let X$\sim$ *Geometric*(θ_1) and Y$\sim$ *Geometric*(θ_2) independently. Then the p.d.f.s of X and Y are as follows,

$$f(x) = \theta_1(1-\theta_1)^{x-1} \qquad x = 0,1,2,3,....$$
$$f(y) = \theta_2(1-\theta_2)^{y-1} \qquad y = 0,1,2,3,.....$$

Let $(r_1, r_2, \ldots, r_m)$ be the first 'm' observations of upper record values from geometric(θ_1). Then, the joint distribution of $(R_1, R_2, \ldots, R_m)$ is given by;

$$P(R_1 = r_1, R_2 = r_2, \ldots, R_m = r_m) = P(R_m = r_m)\prod_{i=1}^{m-1}\frac{P(R_i = r_i)}{P(R_i > r_i)}$$

$$\begin{aligned} P(\underline{R} = \underline{r}) &= \theta_1(1-\theta_1)^{r_m-1}\prod_{i=1}^{m-1}\frac{\theta_1(1-\theta_1)^{r_i-1}}{(1-\theta_1)^{r_i}} \\ &= \theta_1(1-\theta_1)^{r_m-1}(\frac{\theta_1}{1-\theta_1})^{m-1} \\ &= \theta_1^m(1-\theta_1)^{r_m-m} \end{aligned}$$

Similarly, let $(t_1, t_2, \ldots, t_n)$ be the first 'n' observations of upper record values from geometric(θ_2). Then, the joint distribution of $(T_1, T_2, \ldots, T_n)$ is given by;

$$P(T_1 = t_1, T_2 = t_2, \ldots, T_n = t_n) = P(T_n = t_n)\prod_{i=1}^{n-1}\frac{P(T_i = t_i)}{P(T_i > t_i)}$$

$$\begin{aligned}
P(\underline{T} = \underline{t}) &= \theta_2(1-\theta_2)^{t_n-1}\prod_{i=1}^{n-1}\frac{\theta_2(1-\theta_2)^{t_i-1}}{(1-\theta_2)^{t_i}} \\
&= \theta_2(1-\theta_2)^{t_n-1}(\frac{\theta_2}{1-\theta_2})^{n-1} \\
&= \theta_2^n(1-\theta_2)^{t_n-n}
\end{aligned}$$

where by factorization theorem, r_m and t_n are the sufficient statistics for estimating θ_1 and θ_2 respectively.

$$\begin{aligned}
P(R_m = r_m) &= \binom{r_m-1}{m-1}\theta_1^m(1-\theta_1)^{r_m-m}, \qquad r_m = m, m+1, ... \\
P(T_n = t_n) &= \binom{t_n-1}{n-1}\theta_2^n(1-\theta_2)^{t_n-n}, \qquad t_n = n, n+1, ...
\end{aligned}$$

Now, we have assume that $\theta_1 \sim$ Beta(a_1, b_1) and $\theta_2 \sim (a_2, b_2)$ independently,

$$\begin{gathered}
g(\theta_1) = \theta_1^{a_1-1}(1-\theta_1)^{b_1-1}/\beta(a_1, b_1) \\
g(\theta_2) = \theta_2^{a_2-1}(1-\theta_2)^{b_2-1}/\beta(a_2, b_2)
\end{gathered}$$

Therefore, the joint prior distribution of θ_1 and θ_2 is given by;

$$h(\theta_1, \theta_2) = \frac{\theta_1^{a_1-1}(1-\theta_1)^{b_1-1}}{\beta(a_1, b_1)}\frac{\theta_2^{a_2-1}(1-\theta_2)^{b_2-1}}{\beta(a_2, b_2)} \tag{7}$$

The posterior distribution of (θ_1, θ_2) for given (r_m,t_n) is;

$$\pi(\theta_1, \theta_2|r_m, t_n) = \frac{\theta_1^{m+a_1-1}(1-\theta_1)^{r_m-m+b_1-1}}{\beta(m+a_1, r_m-m+b_1)}\frac{\theta_2^{n+a_2-1}(1-\theta_2)^{t_n-n+b_2-1}}{\beta(n+a_2, t_n-n+b_2)} \tag{8}$$

The Bayes estimate of R (stress-strength reliability) is given by;

$$
\begin{aligned}
&\hat{R_{s,k}} = E(R|r_m,t_n)\\
=\ & \int_0^1\int_0^1 \sum_{x=1}^{\infty} P(Y_{s,k}\geq x)P(X=x)\pi(\theta_1,\theta_2|r_m,t_n)d\theta_1 d\theta_2\\
=\ & \int_0^1\int_0^1 \sum_{x=1}^{\infty}\sum_{w=s}^{k}\binom{k}{w}(1-G(x-1))^w G(x-1)^{k-w}P(X=x)\pi(\theta_1,\theta_2|r_m,t_n)d\theta_1 d\theta_2\\
=\ & \int_0^1\int_0^1 \sum_{x=1}^{\infty}\sum_{w=s}^{k}\binom{k}{w}(1-(1-(1-\theta_2)^{x-1})^w(1-(1-\theta_2)^{x-1})^{k-w}\\
& \theta_1(1-\theta_1)^{x-1}\pi(\theta_1,\theta_2|r_m,t_n)d\theta_1 d\theta_2\\
=\ & \int_0^1\int_0^1 \sum_{x=1}^{\infty}\sum_{w=s}^{k}\binom{k}{w}(1-\theta_2)^{(x-1)w}(1-(1-\theta_2)^{x-1})^{k-w}\\
& \theta_1(1-\theta_1)^{x-1}\pi(\theta_1,\theta_2|r_m,t_n)d\theta_1 d\theta_2\\
=\ & \sum_{x=1}^{\infty}\sum_{w=s}^{k}\binom{k}{w}\int_0^1 \theta_1(1-\theta_1)^{x-1}\frac{\theta_1^{m+a_1-1}(1-\theta_1)^{r_m-m+b_1-1}}{\beta(m+a_1,r_m-m+b_1)}d\theta_1\\
& \int_0^1 (1-\theta_2)^{(x-1)w}(1-(1-\theta_2)^{(x-1)})^{k-w}\frac{\theta_2^{n+a_2-1}(1-\theta_2)^{t_n-n+b_2-1}}{\beta(n+a_2,t_n-n+b_2}d\theta_2 \qquad (9)
\end{aligned}
$$

Now, we find differently;

$$
\begin{aligned}
I_{\theta_1} &= \int_0^1 \theta_1(1-\theta_1)^{x-1}\theta_1^{m+a_1+1}(1-\theta_1)^{r_m-m+b_1-1}d\theta_1\\
&= \int_0^1 \theta_1^{m+a_1+2}(1-\theta_1)^{r_m-m+b_1+x-2}d\theta_1\\
&= \beta(m+a_1+1,r_m-m+b_1+x-1) \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
I_{\theta_2} &= \int_0^1 \theta_2^{n+a_2-1}(1-\theta_2)^{t_n-n+b_2-1}(1-\theta_2)^{w(x-1)}(1-(1-\theta_2)^{x-1})^{k-w}d\theta_2\\
&= \int_0^1 (1-\theta_2)^{w(x-1)}\{\binom{k-w}{0}(1-\theta_2)^0-\binom{k-w}{1}(1-\theta_2)^{(x-1)}+\dots\\
&\quad +(-1)^{k-w}\binom{k-w}{k-w}(1-\theta_2)^{(k-w)(x-1)}\theta_2^{n+a_2-1}(1-\theta_2)^{t_n-n+b_2-1}d\theta_2\\
&= \int_0^1 \sum_{i=0}^{k-w}(-1)^u\binom{k-w}{u}\theta_2^{n+a_2-1}(1-\theta_2)^{t_n-n+b_2+(w+u)(x-1)-1}d\theta_2\\
&= \sum_{u=0}^{k-w}(-1)^u\binom{k-w}{u}\beta(n+a_2,t_n-n+b_2+(w+u)(x-1)) \qquad (11)
\end{aligned}
$$

Finally, we found that;

$$\hat{R_{s,k}} = \sum_{x=1}^{\infty} \sum_{w=s}^{k} \binom{k}{w} I_{\theta_1} I_{\theta_2} \tag{12}$$

5. Simulation table

In this section, Monte Carlo simulations have been performed to compare different estimators of $R_{s,k}$. We choose three models, i.e., 1-out-of-3, 3-out-of-3, and 3-out-of-6. We study different upper record values sample sizes $(m, n) \in \{(5,5); (5,7), (7,5), (7,7)\}$. We consider $\theta_1 \in \{0.1, 0.2, 0.5\}$ and $\theta_2 \in \{0.1, 0.3, 0.7\}$. For each model and each combination of $(\theta_1,\ \theta_2)$ we generate a random sample of size (m, n) and calculate UMVUE of $R_{s,k}$. In Bayesian section, we draw random sample of size (m, n) from the prior distribution(s) of the parameter(s) of the stress (strength) distribution(s). The combinations of those hyper-parameters (a_1, b_1, a_2, b_2) of the stress (strength) distribution(s) are so chosen that the expected value of the parameter(s) is(are) equal to the values of $(\theta_1,\ \theta_2)$ taken into consideration; similar to the combinations used in [17]. The procedure has 1000 replications. The tables 1,2 and 3 gives us the average of the point estimates for $R_{1,3}$, $R_{3,3}$ and $R_{3,6}$ under Bayesian set-up and their expected loss respectively, whether in tables 4,5 and 6 we finds the average of the UMVUE of $R_{1,3}$, $R_{3,3}$ and $R_{3,6}$ and their variances respectively. We can see that in almost every case, the UMVUEs as well as the Bayes estimates are closer to the actual value, and their errors are decreasing with the increase of the sample sizes, which verifies the consistency properties of the estimators.

Table 1. Bayes estimates of $R_{1,3}$ and their expected loss

Actual R	a_1	a_2	a_2	b_2	$E(p_1)$	$E(p_2)$	m	n	Estimate	Expected loss
0.918	2	8	1	9	0.2	0.1	5	5	0.85154	0.03676
0.918	2	8	1	9	0.2	0.1	5	7	0.92506	0.01874
0.918	2	8	1	9	0.2	0.1	7	5	0.91076	0.02892
0.918	2	8	1	9	0.2	0.1	7	7	0.92349	0.00909
0.993	5	5	1	9	0.5	0.1	5	5	0.99528	0.00012
0.993	5	5	1	9	0.5	0.1	5	7	0.99966	0.00004
0.993	5	5	1	9	0.5	0.1	7	5	0.99841	0.00040
0.993	5	5	1	9	0.5	0.1	7	5	0.99851	0.00004
0.652	2	8	3	7	0.2	0.3	5	5	0.66869	0.11555
0.652	2	8	3	7	0.2	0.3	5	7	0.74838	0.08137
0.652	2	8	3	7	0.2	0.3	7	5	0.60077	0.10734
0.652	2	8	3	7	0.2	0.3	7	7	0.66192	0.05430
0.924	5	5	3	7	0.5	0.3	5	5	0.93058	0.01200
0.924	5	5	3	7	0.5	0.3	5	7	0.90858	0.01152
0.924	5	5	3	7	0.5	0.3	7	5	0.92243	0.00931
0.924	5	5	3	7	0.5	0.3	7	7	0.92104	0.00653
0.347	2	8	7	3	0.2	0.7	5	5	0.32781	0.06144
0.347	2	8	7	3	0.2	0.7	5	7	0.32993	0.04820
0.347	2	8	7	3	0.2	0.7	7	5	0.33283	0.04284
0.347	2	8	7	3	0.2	0.7	7	7	0.34721	0.03507
0.701	5	5	7	3	0.5	0.7	5	5	0.69174	0.02184
0.701	5	5	7	3	0.5	0.7	5	7	0.68279	0.02182
0.701	5	5	7	3	0.5	0.7	7	5	0.69045	0.02022
0.701	5	5	7	3	0.5	0.7	7	7	0.69918	0.01824

Table 2. Bayes estimates of $R_{1,3}$ and their expected loss

Actual R	a_1	a_2	a_2	b_2	$E(p_1)$	$E(p_2)$	m	n	Estimate	Expected loss
0.48	2	8	1	9	0.2	0.1	5	5	0.59455	0.11681
0.48	2	8	1	9	0.2	0.1	5	7	0.40755	0.09124
0.48	2	8	1	9	0.2	0.1	7	5	0.60607	0.09425
0.48	2	8	1	9	0.2	0.1	7	7	0.48409	0.07789
0.787	5	5	1	9	0.5	0.1	5	5	0.77550	0.03241
0.787	5	5	1	9	0.5	0.1	5	7	0.77859	0.03077
0.787	5	5	1	9	0.5	0.1	7	5	0.78968	0.02529
0.787	5	5	1	9	0.5	0.1	7	7	0.78777	0.01975
0.276	2	8	3	7	0.2	0.3	5	5	0.24378	0.04518
0.276	2	8	3	7	0.2	0.3	5	7	0.26136	0.03919
0.276	2	8	3	7	0.2	0.3	7	5	0.26651	0.02906
0.276	2	8	3	7	0.2	0.3	7	7	0.27524	0.02315
0.604	5	5	3	7	0.5	0.3	5	5	0.58480	0.02833
0.604	5	5	3	7	0.5	0.3	5	7	0.61136	0.02516
0.604	5	5	3	7	0.5	0.3	7	5	0.61533	0.02686
0.604	5	5	3	7	0.5	0.3	7	7	0.60302	0.02334
0.204	2	8	7	3	0.2	0.7	5	5	0.19477	0.01499
0.204	2	8	7	3	0.2	0.7	5	7	0.19946	0.01488
0.204	2	8	7	3	0.2	0.7	7	5	0.01998	0.01457
0.204	2	8	7	3	0.2	0.7	7	7	0.20203	0.01079
0.507	5	5	7	3	0.5	0.7	5	5	0.48632	0.03746
0.507	5	5	7	3	0.5	0.7	5	7	0.51016	0.02903
0.507	5	5	7	3	0.5	0.7	7	5	0.50520	0.02575
0.507	5	5	7	3	0.5	0.7	7	7	0.50790	0.01703

Table 3. Bayes estimates of $R_{3,6}$ and their expected loss

Actual R	a_1	a_2	a_2	b_2	$E(p_1)$	$E(p_2)$	m	n	Estimate	Expected loss
0.48	2	8	1	9	0.2	0.1	5	5	0.83228	0.06317
0.48	2	8	1	9	0.2	0.1	5	7	0.82489	0.05177
0.48	2	8	1	9	0.2	0.1	7	5	0.81793	0.03504
0.48	2	8	1	9	0.2	0.1	7	7	0.82064	0.02348
0.982	5	5	1	9	0.5	0.1	5	5	0.98739	0.00107
0.982	5	5	1	9	0.5	0.1	5	7	0.98708	0.00090
0.982	5	5	1	9	0.5	0.1	7	5	0.98952	0.00185
0.982	5	5	1	9	0.5	0.1	7	7	0.98539	0.00070
0.484	2	8	3	7	0.2	0.3	5	5	0.50075	0.08146
0.484	2	8	3	7	0.2	0.3	5	7	0.46822	0.07569
0.484	2	8	3	7	0.2	0.3	7	5	0.48987	0.07089
0.484	2	8	3	7	0.2	0.3	7	7	0.48857	0.05798
0.839	5	5	3	7	0.5	0.3	5	5	0.84589	0.03894
0.839	5	5	3	7	0.5	0.3	5	7	0.82621	0.02878
0.839	5	5	3	7	0.5	0.3	7	5	0.84836	0.01790
0.839	5	5	3	7	0.5	0.3	7	7	0.84289	0.01778
0.242	2	8	7	3	0.2	0.7	5	5	0.21049	0.02343
0.242	2	8	7	3	0.2	0.7	5	7	0.22511	0.02107
0.242	2	8	7	3	0.2	0.7	7	5	0.24642	0.02020
0.242	2	8	7	3	0.2	0.7	7	7	0.23957	0.01621
0.565	5	5	7	3	0.5	0.7	5	5	0.54666	0.02230
0.565	5	5	7	3	0.5	0.7	5	7	0.58068	0.01435
0.565	5	5	7	3	0.5	0.7	7	5	0.56216	0.01714
0.565	5	5	7	3	0.5	0.7	7	7	0.56811	0.01386

Table 4. UMVUE of $R_{1,3}$ and their Variance

Actual R	p_1	p_2	m	n	Estimate	Variance
0.918	0.2	0.1	5	5	0.90666	0.02119
0.918	0.2	0.1	5	7	0.93510	0.02000
0.918	0.2	0.1	7	5	0.92183	0.02083
0.918	0.2	0.1	7	7	0.91666	0.01716
0.993	0.5	0.1	5	5	0.98500	0.00442
0.993	0.5	0.1	5	7	0.99583	0.00033
0.993	0.5	0.1	7	5	0.99538	0.00021
0.993	0.5	0.1	7	7	0.99238	0.00015
0.652	0.2	0.3	5	5	0.66841	0.08874
0.652	0.2	0.3	5	7	0.67421	0.02868
0.652	0.2	0.3	7	5	0.67502	0.02875
0.652	0.2	0.3	7	7	0.65740	0.01767
0.924	0.5	0.3	5	5	0.91208	0.01618
0.924	0.5	0.3	5	7	0.92900	0.01083
0.924	0.5	0.3	7	5	0.93183	0.01418
0.924	0.5	0.3	7	7	0.92059	0.00755
0.347	0.2	0.7	5	5	0.32417	0.06417
0.347	0.2	0.7	5	7	0.32365	0.06355
0.347	0.2	0.7	7	5	0.32517	0.05924
0.347	0.2	0.7	7	7	0.34261	0.02718
0.701	0.5	0.7	5	5	0.71390	0.04803
0.701	0.5	0.7	5	7	0.69125	0.02502
0.701	0.5	0.7	7	5	0.71710	0.02903
0.701	0.5	0.7	7	7	0.69489	0.02572

Table 5. UMVUE of $R_{3,3}$ and their Variance

Actual R	p_1	p_2	m	n	Estimate	Variance
0.480	0.2	0.1	5	5	0.46827	0.06691
0.480	0.2	0.1	5	7	0.48853	0.02748
0.480	0.2	0.1	7	5	0.48924	0.02713
0.480	0.2	0.1	7	7	0.47926	0.02463
0.787	0.5	0.1	5	5	0.78055	0.04729
0.787	0.5	0.1	5	7	0.78883	0.02694
0.787	0.5	0.1	7	5	0.78394	0.02651
0.787	0.5	0.1	7	7	0.78734	0.01799
0.276	0.2	0.3	5	5	0.26620	0.07042
0.276	0.2	0.3	5	7	0.27434	0.05518
0.276	0.2	0.3	7	5	0.27721	0.04524
0.276	0.2	0.3	10	10	0.27741	0.01159
0.603	0.5	0.3	5	5	0.58123	0.06846
0.603	0.5	0.3	5	7	0.59663	0.06021
0.603	0.5	0.3	7	5	0.59735	0.04816
0.603	0.5	0.3	10	10	0.60543	0.02918
0.204	0.2	0.7	5	5	0.21260	0.05638
0.204	0.2	0.7	5	7	0.18329	0.02695
0.204	0.2	0.7	7	5	0.19263	0.02631
0.204	0.2	0.7	10	10	0.20441	0.01016
0.507	0.5	0.7	5	5	0.49880	0.05937
0.507	0.5	0.7	5	7	0.51743	0.05288
0.507	0.5	0.7	7	5	0.50886	0.04086
0.507	0.5	0.7	7	7	0.50320	0.03624

Table 6. UMVUE of $R_{3,6}$ and their Variance

Actual R	p_1	p_2	m	n	Estimate	Variance
0.822	0.2	0.1	5	5	0.84166	0.04074
0.822	0.2	0.1	5	7	0.83166	0.04060
0.822	0.2	0.1	7	5	0.82750	0.04393
0.822	0.2	0.1	7	7	0.82000	0.02687
0.982	0.5	0.1	5	5	0.96750	0.01029
0.982	0.5	0.1	5	7	0.97500	0.01748
0.982	0.5	0.1	7	5	0.98484	0.00537
0.982	0.5	0.1	7	7	0.98452	0.00319
0.484	0.2	0.3	5	5	0.50754	0.07225
0.484	0.2	0.3	5	7	0.50817	0.06463
0.484	0.2	0.3	7	5	0.47547	0.05664
0.484	0.2	0.3	7	7	0.48768	0.02770
0.839	0.5	0.3	5	5	0.84347	0.07217
0.839	0.5	0.3	5	7	0.83976	0.04137
0.839	0.5	0.3	7	5	0.83541	0.03734
0.839	0.5	0.3	7	7	0.84018	0.02474
0.242	0.2	0.7	5	5	0.23614	0.04216
0.242	0.2	0.7	5	7	0.25416	0.03841
0.242	0.2	0.7	7	5	0.23867	0.03272
0.242	0.2	0.7	7	7	0.24559	0.01979
0.565	0.5	0.7	5	5	0.57643	0.05864
0.565	0.5	0.7	5	7	0.53650	0.05122
0.565	0.5	0.7	7	5	0.55138	0.03425
0.565	0.5	0.7	7	7	0.56281	0.02775

6. Concluding remarks

In this paper, we considered the estimation of strength-stress probability in a multicomponent system, where both stress and strength come from the geometric distribution. We have found the UMVUE and the Bayesian estimate of $R_{s,k}$ using the geometric upper record values in place of general sample observations. The Bayes estimates have been found based on the prior information. The simulation study results slightly favour using upper record values with respect to of the usual random sample. Scientists and practitioners are recommended to use the proposed estimate of $R_{s,k}$ using upper record samples.

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